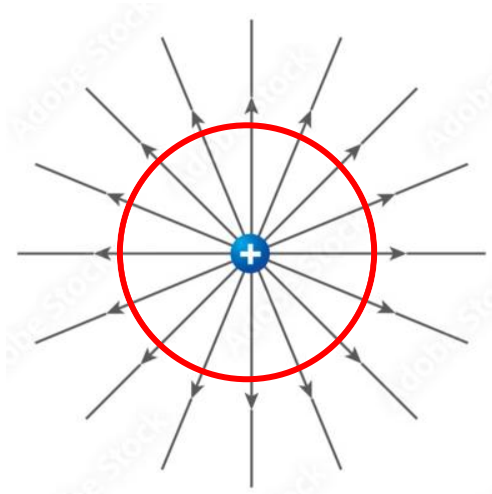
Teacher notes
Topic D

Flux and monopoles.

A single positive charge *Q* is placed inside a sphere of radius *R*, at the center of the sphere.

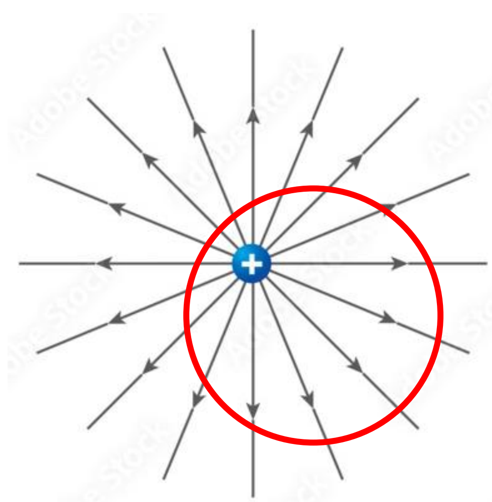


The electric field at the surface of the sphere is $E = \frac{Q}{4\pi\varepsilon_0 R^2}$.

By analogy with magnetic flux, we define electric flux Φ_E as the product $\Phi_E = EA\cos\theta$ where θ is the angle between the normal to the surface of the sphere and the electric field direction. Assuming the normal to the surface of the sphere is outward, the electric flux is

$$\Phi_E = EA\cos\theta = \frac{Q}{4\pi\varepsilon_0 R^2} \times 4\pi R^2 \times \cos 0^\circ = \frac{Q}{\varepsilon_0} = \frac{\text{charge inside sphere}}{\varepsilon_0}$$

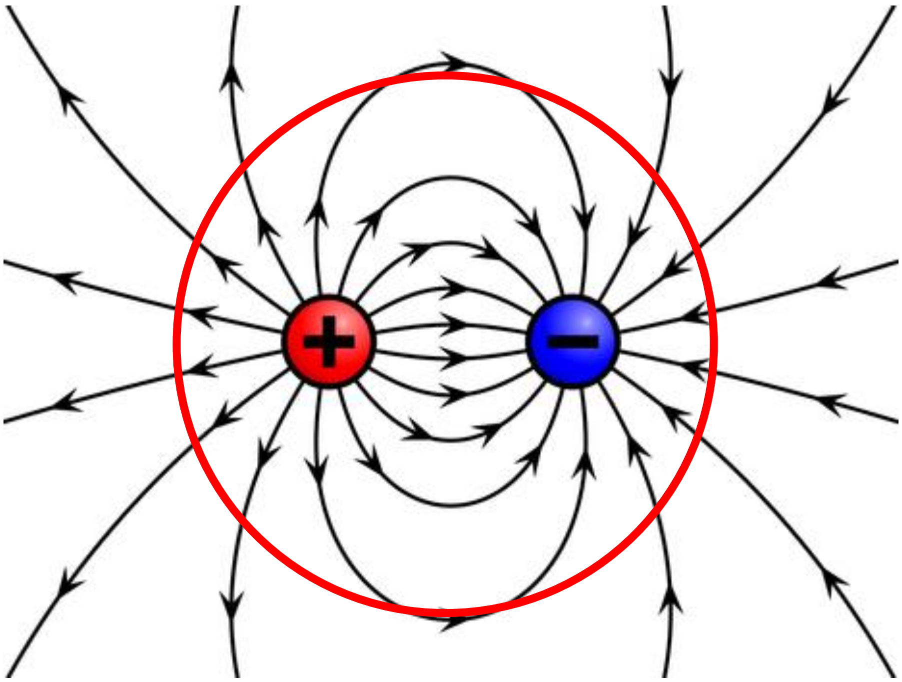
It turns out that the same result holds no matter where inside the sphere the charge is placed.



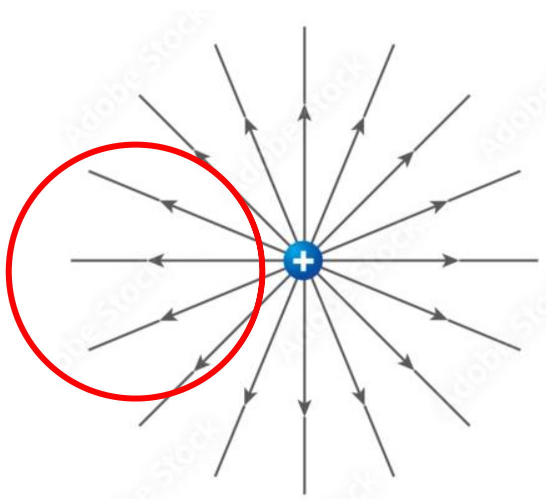
This is a famous result in electricity known as Gauss' theorem:

The electric flux through a closed surface is equal to the total charge inside the surface divided by ε_0.

Using this result, what do you think is the electric flux through a sphere when two equal and opposite charges are placed inside the sphere?



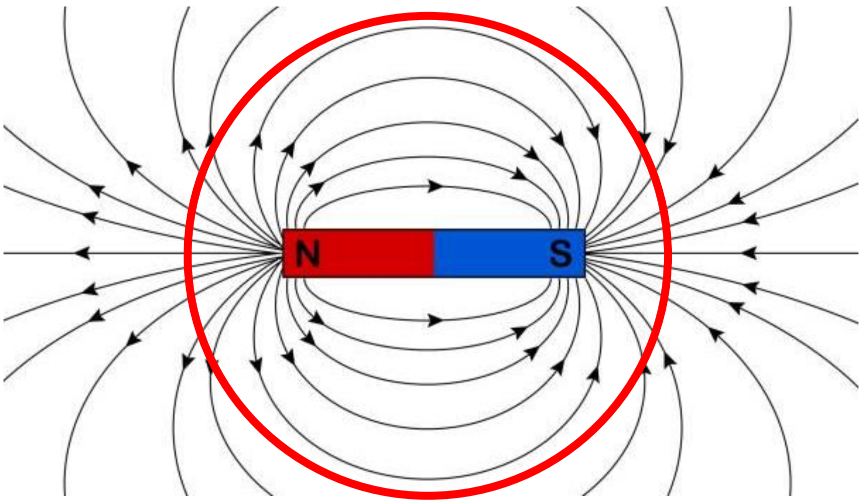
What is the electric flux through a sphere if a charge is placed outside the sphere?



A long time ago it was thought that the north and south poles of a magnet were equivalent to *positive and negative magnetic charges*. A bar magnet was then thought to consist of two equal and opposite magnetic charges. This made sense since the magnetic field pattern of a bar magnet is identical to that of two equal and opposite electric charges (a dipole). The Gauss theorem would then apply to magnetic charges as well with the magnetic flux through a closed surface being proportional to the total magnetic charge inside the surface.

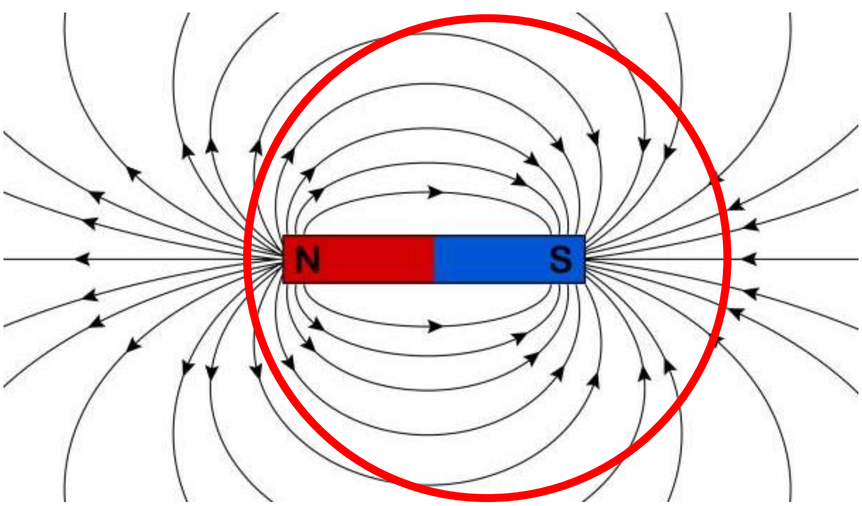
With this in mind:

Imagine that we now place a bar magnet inside a sphere. The center of the magnet coincides with the center of the sphere.

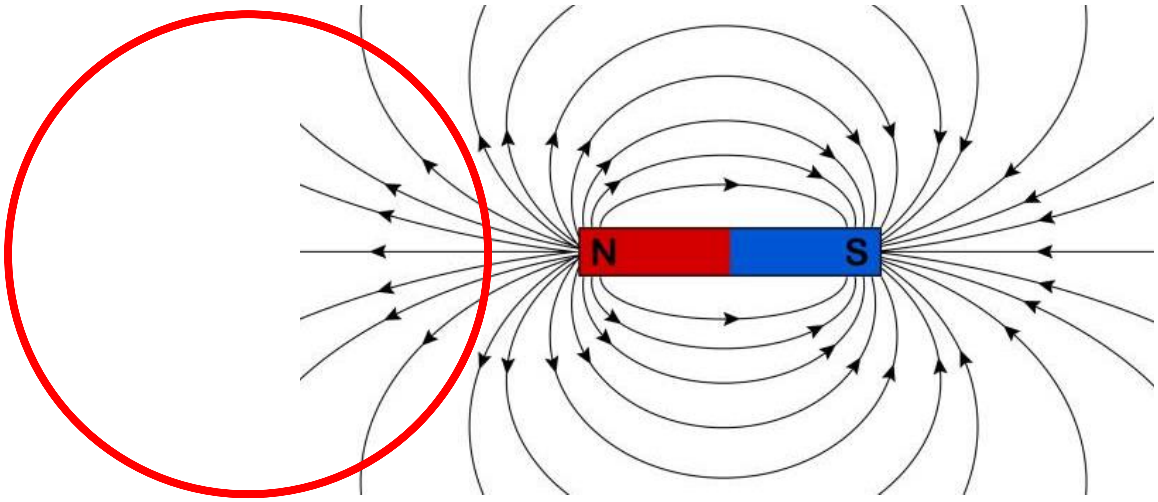


What do you think is the magnetic flux through the sphere?

What happens if the magnet is off center?



What happens if the magnet is outside the sphere?



If magnetic charges existed, we could hope to find particles that have only positive magnetic charge or only negative magnetic charge just as we find this for electric charge in electricity, electrons and protons, say. These particles would be called *magnetic monopoles*.

Theories of particle physics predict the existence of magnetic monopoles but a search for magnetic monopoles spanning more than four decades has not revealed any.

We know that electric charge is quantized, that is all electric charges are an integer multiple of a fundamental unit which happens to be the charge of the electron (or one third of this if we take quarks into account). But what is the reason for this fundamental observation? Why do we not expect to ever find a particle whose electric charge is anything other than an integer multiple of e?

In 1931, Paul Dirac, one of the greatest physicists of the 20th century, provided a simple answer based on quantum mechanics. In about three lines, he proved that even if a single magnetic monopole existed in the Universe, that would imply quantization of electric charge. But these three lines are not part of high school physics so I will omit them and I encourage the reader to look into this amazing result further!